INFLUENCE OF HEAT TRANSFER ON MAGNETOHYDRODYNAMICS FLUID PAST AN INFINITE VERTICAL POROUS PLATE WITH SUCTION

by
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Abstract
The influence of heat transfer on magnetohydrodynamic (MHD) fluid flow past a vertical porous plate with variable suction is studied. The nonlinear partial differential equation models were solved using the Crank-Nikolson finite difference scheme. A parametric study was performed to illustrate the impact of Ercket, Radiation, Grashof, Porosity, Magnetic, Prandtl and Heat source parameters on the velocity profile, skin friction and Nusset number.

Keywords- Magnetohydrodynamics, Ercket, Grashof, Prandtl, Nusset number, Skin friction.

1 INTRODUCTION
The study of unsteady natural convective flow of viscous incompressible fluid past vertical bodies in the presence of magnetic has a wide engineering and technological applications. Natural or Free convection is the principal mode of heat transfer from pipes, refrigerating coils, hot radiators etc. The movement of fluid in free convection is due to the fact that the fluid particles in the immediate vicinity of the hot object become warmer than the surrounding fluid resulting in a local change of density. The warmer fluid would be replaced by the colder fluid creating convection currents. These currents originate when a body force (gravitational, centrifugal, electrostatic etc.) acts on a fluid in which there are density gradients. The force which induces these convection currents is called a buoyancy force which is due to the presence of a density gradient within the fluid and a body force.

Many studies have been carried on this subject. Sharma (2008) studied the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non-conducting plate with variable suction and exponentially decaying heat generation in the presence of transverse magnetic field. The governing equations of motion and energy were transformed into ordinary differential equations using time dependent similarity technique and the resulting equations were solved under the prescribed boundary conditions using Runge-Kutta fourth order technique along with shooting technique. The variations of flow velocity, temperature, skin-friction and rate of heat transfer characteristics with various parameters like Grashof (Gr), Prandtl (Pr), Magnetic (M) were presented. It was observed that heat source and suction affects the boundary layer thicknesses, velocity and temperature profiles and skin-friction and rate of heat transfer at the plate surface in such flows.

Makinde, (2011). Investigated Second Law Analysis for Variable Viscosity Hydromagnetic Boundary layer Flow with Thermal Radiation and Newtonian Heating. The main focus is on the entropy generation characteristics for hydromagnetic boundary layer flow under the influence of thermal radiation and Newtonian heating at the plate surface. Using local similarity solution technique and shooting quadrature, the velocity and temperature profiles are obtained numerically and utilized to compute the entropy generation number. The effects of magnetic field parameter, Brinkmann number, the Prandtl number, variable viscosity parameter, radiation parameter and local Biot number on the fluid velocity
profiles, temperature profiles, local skin friction and local Nusselt number are presented. The influences of the same parameters and the dimensionless group parameter on the entropy generation rate in the flow regime and Bejan number were computed and shown graphically. It was observed that the peak of entropy generation rate is attained within the boundary layer region and plate surface act as a strong source of entropy generation and heat transfer irreversibility.

Dada et al. (2014) investigated the unsteady free convection of magneto-hydrodynamic radiative fluid flow past an infinite vertical porous plate in a porous medium under the limit of an optically thin fluid is investigated. An implicit finite differences scheme of crank-Nicolson type was employed in solving the resulting coupled non-linear partial differential equations and obtained the dimensionless velocity and temperature. Velocity and temperature fields were studied and their profiles and Nusselt number are given. They observed, among others, that the radiation parameter decreases the velocity and temperature of fluid while the suction/injection parameter leads to an increase in the transient velocity and temperature of fluid. Also, a rise in magnetic field and permeability parameter parameters reduced the velocity significantly and temperature slightly.

The present work is a follow up on the work of Dada et al. (2014). He neglected the Grashof Gr and Heat absorbent S parameter effects on the fluid. Grashof number is the dimensionless parameter that governs the fluid flow. This work with the influence of heat transfer on magnetohydrodynamic fluid flow past an infinite vertical porous plate with Grashof and Heat absorbent number is presented in this study. The momentum equation includes the Grashof parameter while the Heat absorbent is in the energy equation.

2.0. FORMULATION OF THE PROBLEM
Consider unsteady two-dimensional hydromagnetic laminar, incompressible, viscous, electrically conducting and heat source past an infinite vertical porous plate in the presence of thermal diffusion and thermal radiation effects. According to the coordinate system, the $x^*$-axis is taken along the plate in upward direction and $y^*$-axis is normal to the plate. The fluid is assumed to be a gray, absorbing-emitting but non-scattering medium. Initially, at time $t = 0$, the plate and the fluid are assumed to be at the same temperature $T_\infty$ and stationary. At time $t > 0$, the temperature of the plate is raised or lowered to $T_w$. The flow is considered to be laminar without any pressure gradient in the flow direction. The radiative heat flux in the $x^*$-direction is considered negligible in comparison with that in the $y$-direction. Theradiative heat flux in $y^*$-direction is only considered. All variables are function of $t^*$ and $y^*$ only. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetics Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is the derived form an order of magnitude analysis of the full Naiver-stokes equation. It is assumed here that the hole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. The magnetic field, viscous permeability, viscous dissipation and radiation are important in flow problem hence they are assumed to play important roles. It is also assumed that the fluid properties are constant except for the body force terms in the momentum equation, which are approximated by the Boussinesq relation. With these assumptions, the flow can be governed by the momentum, continuity and energy equations.

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3.0. Governing Equation

The governing equations of the model in dimensional form are

\[
\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{v^*}{1 + \alpha_k} \frac{\partial^2 u^*}{\partial y^*^2} + g \beta \left( T^* - T_\infty^* \right) - \frac{\sigma B_0}{\rho} u^* v^* k^* \tag{1}
\]

\[
\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T^*}{\partial y^*^2} \right) + \frac{1}{\rho c_p} \frac{\partial q_T}{\partial y^*} + \frac{Q_0}{\rho c_p} \left( T^* - T_\infty^* \right) \tag{2}
\]

The boundary conditions for the velocity and temperature fields are

\[u^* = 0, \quad T^* = T_\infty^* \quad \text{for all} \quad y^* \]

\[t^* > 0, \quad u^* = 0, \quad T^* \to T_w^* \quad \text{as} \quad y^* = 0 \]

\[u^* = 0, \quad T^* \to T_\infty^* \quad \text{as} \quad y^* \to \infty \tag{3}\]

The above equations (1) and (2) in dimensionless form are

\[
\frac{\partial u}{\partial t} + v \frac{\partial T}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \left( M + K \right) u + Gr \theta \tag{4}
\]

\[
\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial y^2} \right) + \frac{Ec}{Pr} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{Pr} \left( R - S \right) \theta \tag{5}
\]

With the boundary conditions

\[u = 0, \quad \theta = 1, \quad \text{for} \quad y = 0 \]

\[u = 0, \quad \theta = 0, \quad \text{as} \quad y \to \infty \tag{6}\]

3.1. Introducing Skin friction and Nusselt number

The important characteristics of the problem are the Skin friction and Nusselt number.

**Skin friction**: The dimensions shearing stress on the surface body, due to a fluid motion is known as Skin friction \((c_f)\) and is defined by the newton law of velocity.

\[
\tau_{\omega} = \mu \frac{\partial u^*}{\partial y^*} \tag{7}
\]

and the coefficient of skin friction at the plate is given by:

\[
c_f = \left[ \frac{\tau_{\omega}}{\mu v_0} \right] \left[ \frac{\partial u}{\partial x} \right] \tag{8}
\]

**Nusselt number**: The dimensionless coefficient of heat transfer is known as Nusselt number \((Nu)\). Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer \(q_{\infty}\).

\[
q_{\infty} = -k \left( \frac{\partial T^*}{\partial y^*} \right)_{y=0} - \frac{\partial \sigma^*}{\partial k T^*} \left( \frac{\partial T^*}{\partial y^*} \right)_{y=0} \tag{9}
\]

The dimensionless forms

\[
q_{\infty} = -k V_0 \left( \frac{T_w^* - T_\infty^*}{v} \right) \left( 1 + \frac{4R}{3} \right) \left( 1 + \frac{\partial \theta}{\partial y} \right)_{y=0} \tag{10}
\]

\[
X = \frac{x v_0}{v}, \quad y = \frac{v_0 y^*}{v}, \quad T^* = (T_\infty^* - T_w^*), \quad Nu = \frac{x q_{\infty}}{k (T_w^* - T_\infty^*)} \Rightarrow Nu = \frac{x q_{\infty}}{k (T_w^* - T_\infty^*)} \tag{11}
\]

\[
Nu R_e^{-1} = - \left( 1 + \frac{4R}{3} \right) \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \tag{12}
\]

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Where $Re = \frac{v_0L}{v}$ is the Reynolds number.

The rate of heat transfer in terms of nusselt number at the plate is given by

$Nu = \left[\frac{\partial \theta}{\partial x}\right]_{x=0}$ (12)

## 4.0. Numerical Solution

We now apply the Crank - Nicolson implicit finite difference method which is known to be convergent and stable. This method is used to solve Equations (3) and (4) subject to the boundary conditions given in equation (5).

### 4.1 Momentum Equation (Dimensionless)

Applying on the momentum equation and evaluating, we have

$$
\frac{\partial \hat{u}}{\partial \hat{t}} + \frac{\partial \hat{u}}{\partial \hat{y}} = \hat{\theta} \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} - (M + K) \hat{u} + Gr \hat{\theta}
$$

$$
\left( \frac{U_j^{k+1} - U_j^k}{\Delta t}\right) + \left( \frac{U_{j+1}^{k+1} - U_{j+1}^k + U_{j-1}^{k+1} - U_{j-1}^k}{4\Delta y} \right) = \left( \frac{U_{j+1}^{k+1} - 2U_{j}^{k+1} + U_{j-1}^{k+1} - 4U_{j}^{k+1} + U_{j}^{k+1}}{2(\Delta y)^2} \right)
$$

$$
-L\left( \frac{U_{j+1}^{k+1} + U_{j}^k}{2} \right) + Gr \left( \frac{\theta_{j+1}^{k+1} + \theta_j^k}{2} \right)
$$

Which can be written in matrix formas

$$
\begin{bmatrix}
A_1 & A_2 & 0 & 0 & 0 & U_{j+1}^{k+1} & b_1 \\
A_1 & A_2 & A_3 & 0 & 0 & U_{j}^{k+1} & b_2 \\
0 & A_1 & A_2 & A_3 & 0 & U_{j+1}^{k+1} & b_3 \\
\end{bmatrix}
$$

Where $A_1 = vr_i + 2r_2, A_2 = 4 + 4r_2 + r_3L_1, A_3 = vr_i - 2r_2, A_4 = 4 - 4r_2 - r_3L_1, A_5 = r_3Gr
$

$$
\frac{\Delta t}{\Delta y}, r_2 = \frac{\Delta t}{\Delta y}, r_3 = 2\Delta t, L_1 = (M + K)
$$

$$
b_j = A_jU_{j+1}^k + A_jU_{j}^k + A_j(\theta_{j+1}^{k+1} - \theta_{j}^k)
$$

(15)

### 4.2 Energy Equation (Dimensionless)

Discretise equation (5) and re-arrange to obtain

$$
\frac{(\theta_{j+1}^{k+1} - \theta_j^k) \text{Pr} - v}{\Delta t} \left( \frac{\theta_{j+1}^{k+1} - \theta_{j+1}^{k+1} + \theta_{j}^{k+1} - \theta_{j}^{k+1}}{4\Delta y} \right) = \left( \frac{\theta_{j+1}^{k+1} - 2\theta_{j+1}^{k+1} + \theta_{j+1}^{k+1} - \theta_{j+1}^{k+1} - 2\theta_{j}^{k} + \theta_{j}^{k}}{2(\Delta y)^2} \right)
$$

$$
+ \text{Pr} Ec \left( \frac{U_{j+1}^{k+1} - U_{j}^k}{2} \right)^2 - (R - S) \left( \frac{\theta_{j+1}^{k+1} + \theta_j^k}{2} \right)
$$

This gives

$$
-B_1(\theta_{j+1}^{k+1} + B_2\theta_j^{k+1} + B_3\theta_{j+1}^k) = B_1\theta_{j+1}^k + B_4\theta_j^{k+1} - B_5\theta_{j+1}^{k+1} + B_5U_{j+1}^{k+1} - U_{j+1}^k
$$

(17)

where
In matrix form we have
\[
\begin{bmatrix}
B_2 & B_3 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\phi_{1}^{i+1}
\phi_{2}^{i+1}
\phi_{3}^{i+1}
\phi_{4}^{i+1}
\phi_{5}^{i+1}
\end{bmatrix}
= \begin{bmatrix}
p_1
p_2
p_3
p_4
p_5
\end{bmatrix}
\]
(19)

\[
p_j = B_j \theta_j^k + B_j \theta_j^{k+1} - B_j \theta_j^{k+1} + B_j \left(U_j^{k+1} - U_j^{k+1}\right)
\]
(20)

The finite differences representation at every internal nodal point on a particular k-level constitute a tridiagonal system of equations, the above equations are solved using Thomas Algorithm. The values of \(u\) and \(\theta\) are known at all grid points at the initial time \(t = 0\). The values of \(u\) and \(\theta\) at time level \((k+1)\) using the known values at previous time level \(k\) are then calculated.

Hence the implicit Crank Nicolson method is unconditionally stable and therefore consistency and stability ensures the convergence of the scheme. The computations are carried out for different values of physical parameters involved in the problem.

We compute the velocity and temperature of the fluid using the dimensionless equations (4) and (5) with boundary condition 6 (Taking \(N = 4, \Delta t = 0.01, V = 0.3, Pr = 0.71, R = 3, S = 0, M = 1, K = 3, Ec = 0.1, Gr = 1\))

Applying the Crank Nicholson method we obtain
\[
\frac{\partial \theta}{\partial t} + \nu \frac{\partial^2 T}{\partial y^2} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + \frac{1}{Pr} (R - S) \theta
\]

\[
- B_j \theta_j^{i+1} + B_j \theta_j^{i+1} + B_j \theta_j^{i+1} + B_j \theta_j^{i+1} + B_j \left(U_j^{i+1} - U_j^{i+1}\right)
\]

Where
\[
r_1 = \frac{\Delta t}{\Delta y}, r_2 = \frac{\Delta t}{\Delta y}, r_3 = 2 \Delta t, r_4 = r_3 (R - S), B_1 = (\nu \Delta t Pr + 2 r_2), B_2 = 4 Pr + 4 r_2 - r_3,
\]

\[
B_3 = \nu Pr r_1 - 2 r_2, B_4 = 4 Pr - 4 r_2 + r_3, B_5 = r_3 Pr Ec
\]
(20)

Now, we set \(k = 0\) and \(j = 1, 2, \ldots, N\) to obtain a system of equations which can be written

In matrix form as
\[
\begin{bmatrix}
b_1 & d_1 & 0 & 0 & \theta_1
a_2 & b_2 & d_2 & 0 & \theta_2
0 & a_3 & b_3 & d_3 & \theta_3
0 & 0 & a_4 & b_4 & \theta_4
\end{bmatrix}
\begin{bmatrix}
p_1
p_2
p_3
p_4
\end{bmatrix}
\]
(21)

where \(B_1 = a's, B_2 = b's, B_3 = d's\)

From boundary condition
\[
\theta_0^0 = 1, \theta_0^1 = 1.16, \theta_0^2 = 1.32, \theta_0^3 = 1.48, \theta_0^4 = 0, \theta_0^5 = ?, \theta_0^6 = 0.01
\]
(22)

Introducing dimensionless parameter Nusselt (Nu) to obtain \(\theta_S\), we have
\[
Nu = -\frac{\partial \theta}{\partial y}\bigg|_{y=0}
\]
(23)
Discretize equation (23) using backward approximation to obtain

$$0 = \frac{-3\theta_{j+1} - 4\theta_j + \theta_{j-1}}{2\Delta y}, \text{ from Shafigh M}$$

(24)

Putting (24) into equation (20) for j = 4. We have

$$\left( \frac{B_3}{3} - B_1 \right) \theta_1 + \left( B_2 - \frac{4B_1}{3} \right) \theta_2 = \left( B_3 - \frac{B_1}{3} \right) \theta_3 + \left( B_4 + \frac{4B_1}{3} \right) \theta_4 + B_5 \left( U_5^0 - U_5^* \right)$$

(25)

Applying equation 3.135 and boundary condition and solving, we have

$$\begin{bmatrix}
4.3425 & -0.767938 & 0 & 0 \\
-0.794563 & 4.3425 & -0.767938 & 0 \\
0 & -0.794563 & 4.3425 & -0.767938 \\
0 & 0 & 0 & -1.054167
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4
\end{bmatrix} = 3.367833, 3.823887, 3.028469, 1.554494$$

(26)

We solve equation (26) using Thomas Algorithm to obtain the values of the temperature, i.e. Nusselt values which are

$$\theta_1 = 0.9945, \theta_2 = 1.2382, \theta_3 = 0.9930, \theta_4 = 0.3906$$

Now to obtain the velocity, we solve

$$-A_1 U_{j-1}^{k+1} + A_2 U_j^{k+1} + A_3 U_{j+1}^{k+1} = A_1 U_{j-1}^k + A_2 U_j^k - A_4 U_{j+1}^k + A_5 \left( \theta_j^{k+1} + \theta_j^k \right)$$

(27)

Where

$$A_1 = \nu r_1, A_2 = 4 + 4r_2 + r_3 l_1, A_3 = \nu r_1 - 2r_2, A_4 = 4 - 4r_2 - r_3 l_1, A_5 = r_3 G_r$$

(28)

When k=0, then j=1, 2…N we have a system of equations in matrix form gives

$$\begin{bmatrix}
b_1 & d_1 & 0 & 0 \\
b_2 & d_2 & 0 & 0 \\
0 & a_1 & b_1 & d_2 \\
0 & 0 & a_1 & b_1
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} =
\begin{bmatrix}
g_1 \\
g_2 \\
g_3 \\
g_4
\end{bmatrix}$$

(29)

Where A_1 = a, A_2 = b, A_3 = d

Introducing dimensionless parameter Skin friction to obtain U_5, we have

$$C_F = \frac{du}{dy} \bigg|_{y=0}$$

$$0 = \frac{-3U_{j+1} + 4U_j + U_{j-1}}{2\Delta y}$$

(30)

Therefore

$$a_{*4} = \left( A_3 - \frac{A_1}{3} \right) \text{ and } b_{*4} = \left( A_2 - \frac{4A_3}{3} \right)$$

(31)

we have the following values

$$U_0^0 = 0, U_1^0 = 0.16, U_2^0 = 0.32, U_3^0 = 0.48, U_4^0 = 0, U_5^0 = 0.01, r_1 = 0.0625, r_2 = 0.390625, r_3 = 0.02, l_1 = 4$$

Hence we form the matrix

$$\begin{bmatrix}
5.6425 & -0.7625 & 0 & 0 \\
-0.8 & 5.6425 & -0.7625 & 0 \\
0 & -0.8 & 5.6425 & -0.7625 \\
0 & 0 & -1.054167 & 6.659167
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{bmatrix} = 0.67329, 1.253516, 1.43706, 0.513812$$

(32)

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Solving with Thomas Algorithm of Gaussian elimination method using matlab, we obtain the values of velocity as 
\[ U_1 = 0.0924, \quad U_2 = 0.1992, \quad U_3 = 0.2668, \quad U_4 = 0.1194 \]

5.0. Results and Discussion.

Computations are carried out for velocity profiles, temperature profiles, Skin –friction \( (C_f) \) and Nusselt Number \( (Nu) \) for various value of the fluid parameters Grashof \( (Gr) \), Radiation \( (R) \), Permeability \( (K) \), Suction\( (V) \), Prandl number\( (Pr) \), Eckert number\( (Ec) \), Magnetic \( (M) \) and Heat Source \( (S) \) which are varied in order to account for their effects. We present numerical results for velocity and temperature profiles on graphs while the Skin-friction coefficient \( (C_f) \) and Nusselt number \( (Nu) \) are shown in tabular form.

For the purpose of the numerical computation, we adapt \( R = 3, \ K = 3, \ Pr = 0.71, \ V = 0.3, \ Ec = 0.1, \ H = 1 \) from [3] and values like \( Gr = 1, \ S = 1 \) from [8].

### Table 1

<table>
<thead>
<tr>
<th>Fluid Parameter</th>
<th>For Momentum Equation</th>
<th></th>
<th>For Energy Equation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Skin Friction</td>
<td>Nusselt</td>
<td>Skin Friction</td>
<td>Nusselt</td>
</tr>
<tr>
<td>Gr = 1</td>
<td>0.4034</td>
<td>2.9290</td>
<td>Pr = 0.71</td>
<td>0.2653</td>
</tr>
<tr>
<td>Gr = 2</td>
<td>0.8181</td>
<td>2.9130</td>
<td>Pr = 1.00</td>
<td>0.2719</td>
</tr>
<tr>
<td>Gr = 3</td>
<td>1.2305</td>
<td>2.8862</td>
<td>Pr = 0.70</td>
<td>0.2767</td>
</tr>
<tr>
<td>Gr = 4</td>
<td>1.6472</td>
<td>2.8481</td>
<td>Pr = 15.00</td>
<td>0.2897</td>
</tr>
<tr>
<td>K = 0.5</td>
<td>0.4403</td>
<td>2.9278</td>
<td>R = 3</td>
<td>0.2653</td>
</tr>
<tr>
<td>K = 1.0</td>
<td>0.4084</td>
<td>2.9290</td>
<td>R = 4</td>
<td>0.2340</td>
</tr>
<tr>
<td>K = 1.5</td>
<td>0.3822</td>
<td>2.9299</td>
<td>R = 5</td>
<td>0.2120</td>
</tr>
<tr>
<td>K = 2.0</td>
<td>0.3605</td>
<td>2.9305</td>
<td>R = 6</td>
<td>0.1952</td>
</tr>
<tr>
<td>V = 0.1</td>
<td>0.4142</td>
<td>3.0451</td>
<td>Ec = 20</td>
<td>0.2744</td>
</tr>
<tr>
<td>V = 0.2</td>
<td>0.4115</td>
<td>2.9852</td>
<td>Ec = 40</td>
<td>0.2855</td>
</tr>
<tr>
<td>V = 0.3</td>
<td>0.4084</td>
<td>2.9290</td>
<td>Ec = 60</td>
<td>0.2987</td>
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<tr>
<td>V = 0.4</td>
<td>0.4046</td>
<td>2.8765</td>
<td>Ec = 80</td>
<td>0.3147</td>
</tr>
<tr>
<td>H = 1</td>
<td>0.4084</td>
<td>2.9290</td>
<td>V = 0.2</td>
<td>0.2757</td>
</tr>
<tr>
<td>H = 2</td>
<td>0.3605</td>
<td>2.9305</td>
<td>V = 0.4</td>
<td>0.2729</td>
</tr>
<tr>
<td>H = 3</td>
<td>0.3261</td>
<td>2.9315</td>
<td>V = 0.6</td>
<td>0.2692</td>
</tr>
<tr>
<td>H = 4</td>
<td>0.3000</td>
<td>2.9321</td>
<td>V = 0.8</td>
<td>0.2647</td>
</tr>
</tbody>
</table>

\( (R = 3, \ K = 3, \ Pr = 0.71, \ V = 0.3, \ Ec = 0.1, \ H = 1, \ Gr = 1, \ S = 1) \)

The influence of various parameters on Skin-friction coefficient and Nusselt number are shown in Table 1.

In order to show the influence of all the parameters involved on the flow, a selected set of graphical results are presented in figures.
Figure 1  Effect of different values of Grashof (Gr) on velocity profile

Figure 2  Effect of different values of Permeability (K) on velocity profile

Figure 3: Effect of different values of Magnet (H) on velocity profile

Figure 4: Effect of different values of Suction on velocity profile

From Table 2, it is observed that an increase in Ec and S leads to a rise in the skin-friction coefficient and Nusselt number, respectively, while an increase in R leads to a fall in the skin-friction coefficient and Nusselt number, respectively. It is also seen that as Pr increases there is a rise in the skin-friction coefficient and a fall in Nusselt numbers, respectively. The suction V, fall in the skin-friction coefficient and a rise in Nusselt numbers as the parameter increases.

Figure 5: Effect of different values of Prandtl (Pr) on Velocity profile
Figure 6: Effect of different values of Prandtl (Pr) on Temperature profile

Figure 7: Effect of different values of Radiation (R) on Velocity profile

Figure 8: Effect of different values of Radiation (R) on Temperature profile

Figure 9: Effect of different values of Eckert (Ec) on Temperature profile

Figure 10: Effect of different values of Suction (V) on Temperature profile
Figure 11: Effect of different values of Suction(V) on Velocity profile

Figure 12: Effect of different values of Heat Source (S) on Temperature profile

6.0. Discussion of Results

The influence of thermal Grashof number (Gr), on the velocity is shown in figure 1. It is observed that an increasing in Gr leads to a rise in the values of velocity and temperature. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity. Figure 2 shows that the permeability (k) of porous medium, parameter (k) increases the resistance of the porous medium, thus decreasing the velocity of the flow. So an increase in permeability k causes a drop in the velocity of fluid.

In figure 3, we observe that the effect of magnetic parameter (H) is to retard the main velocity of the flow field due to the magnetic pull of the Lorentz force acting on the flow field. As the values of the magnetic parameter (H) increases, velocity u decreases.

From figure 4 in the presence of growing suction, the velocity of the flow field is found to decrease. Also an increase in suction parameters reduces the skin-friction at the wall.

From Figure 6 and 7, it is noticed that velocity decreases with the increase in Prandtl number (Pr). It is observed that reducing the Prandtl number produces significant increase in the velocity condition of the fluid. Also in figure 6, it is observed that increasing the Prandtl number produces significant decrease in the thermal condition of the fluid. Thermal conductivity is accelerated with small values of prandtl number thus causing rapid diffusion of heat for smaller Prandtl number than for higher values of Prandtl number.

In figures 8 and 9 it is observed that the fluid pressure decreases as the radiation parameter increases on the velocity profiles. Also, it is noticed that the radiation parameter (N) increases the temperature profiles decreases.

In figure 10, it is observed that growing viscous dissipative (Eckert number) causes an increase in the velocity profiles and temperature profiles across the boundary layer.

In figures 11 and 12, the suction parameter leads to increase in the transient velocity and temperature of fluid, that is the velocity and temperature of fluid increases with increasing the suction parameter.

7.0. Conclusions

Having studied influence of heat transfer on magnetohydrodynamic fluid flow past a vertical porous plate with suction, we conclude as follows:

1. An increase in Grashof number leads to an increase in the heavy flow of fluid velocity owing to the increase in quality of buoyancy force. The highest point of the velocity goes up quickly near the porous plate as buoyancy force for heat movement rises while the temperature reduces.

2. The increase in Hartmann number decreases both velocity and temperature of the fluid. This is due to the fact that the effect of magnetic parameter (H) is to retard the main velocity of the flow field due to the magnetic pull of the Lorentz force acting on the flow field. As the values of the magnetic parameter (H) increases, velocity u decreases.
3. The increase in the permeability (K) number leads to a decrease in velocity. This is because the effect of the dimensionless porous medium K becomes smaller as K increases. Physically, this result can be achieved when the holes of the porous medium may be neglected and increases in temperature.

4. The increase in values of Eckert number, heat source, and suction brings increases in velocity and temperature distributions.

5. An increase in the Prandtl number decreases the velocity and temperature. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl number as the thermal boundary layer is thicker and the rate of heat transfer is reduced.

6. The increase in thermal Grashof number increases the skin – friction.

7. An increase in the Hartmann number and Prandtl number decreases the skin – friction.

8. An increase in the Prandtl number decreases the Nusselt number.

References
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